3/7/17 "Too many of us are not living our dreams because we are living our fears."-Les Brown

HW: "Systems of Linear Equations" Homework #1-4
Test 2 on Tuesday 3/14

AIM: How do we solve a system of equations?

Warm Up:

Solve the following systems of equations algebraically by either substitution or elimination.

2 by 2 Equations (2 eq. w/ 2 variables)
3 by 3 Equations (3 eq. w/ 3 variables)

1. +

$$2x - y = -1$$
$$2x + y = -7$$

$$4x = -8$$
$$\frac{4x}{4} = \frac{-8}{4}$$
$$x = -2$$

To find y:

$$2(-2) - y = -1$$
$$-4 - y = -1$$
$$+4 \qquad +4$$
$$-y = 3$$
$$\frac{-y}{-1} = \frac{3}{-1}$$
$$y = -3$$

$$\boxed{\begin{matrix} x = -2 \\ y = -3 \\ \text{OR} \\ (-2, -3) \end{matrix}}$$

substitution - use if an equation says "$y=$" or "$x=$"

2. $2x + 2y = 3$
$x = 4y - 1$

$\rightarrow 2(4y-1) + 2y = 3$
$8y - 2 + 2y = 3$
$10y - 2 = 3$
$10y = 5$
$y = \frac{5}{10} = \frac{1}{2}$

Find x:
$x = 4(\frac{1}{2}) - 1$
$x = 2 - 1$
$x = 1$

$(1, \frac{1}{2})$
OR
$x = 1$
$y = \frac{1}{2}$

3. $2(x - 2y = 3)$
$-2x + 4y = 1$

$\oplus$ $\cancel{2x} - \cancel{4y} = 6$
$\cancel{-2x} + \cancel{4y} = 1$
$0 = 7$ F

To get what we want — we need to do all (BALANCE)

Lines are parallel

Does not make sense therefore NO SOLUTION

4. $-2(2x - y = 1) \rightarrow$
$4x - 2y = 2$

$\oplus$ $-4x + 2y = -2$
$4x - 2y = 2$
$0 + 0 = 0$
$0 = 0$ T

Infinite Solutions

(The equations are the same line)

5. $5(3x + 2y = 2) \rightarrow$
$-3(5x + 7y = -4) \rightarrow$

$\oplus$ $\cancel{15x} + 10y = 10$
$\cancel{-15x} - 21y = 12$
$\frac{-11y}{-11} = \frac{22}{-11}$
$y = -2$

Find x:
$3x + 2(-2) = 2$
$3x - 4 = 2$
$+4 \quad +4$
$\frac{3x}{3} = \frac{6}{3}$ $x = 2$

$\boxed{x=2,\ y=-2 \text{ OR } (2,-2)}$

6. $3x + 2y = -9$
$-2(2x + y = -7) \rightarrow$

$3x + \cancel{2y} = -9$
$-4x - \cancel{2y} = 14$
$\frac{-1x}{-1} = \frac{5}{-1}$
$x = -5$

To find y:
$3(-5) + 2y = -9$
$-15 + 2y = -9$
$+15 \quad +15$
$\frac{2y}{2} = \frac{6}{2}$
$y = 3$

$\boxed{(-5, 3)}$

Summary

1) One solution

2) Infinite solutions (equations are the same line)

3) No Solution (Equations are Parallel lines)

You should be very familiar with solving two-by-two systems of linear equations (two equations and two unknowns). In this lesson, we will extend the method of **elimination** to linear systems of three equations and three unknowns. These linear systems serve as the basis for a field of math known as **Linear Algebra**.

7. Consider the three-by-three system of linear equations shown below. Each equation is numbered in this first exercise to help keep track of our manipulations.

$$\begin{aligned} &(1) \quad 2x + y + z = 15 \\ &(2) \quad 6x - 3y - z = 35 \\ &(3) \quad -4x + 4y - z = -14 \end{aligned}$$

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?

created 2 equations with only 2 variables

$$\begin{aligned} &① \quad 2x + y \cancel{+ z} = 15 \\ ⊕\ &② \quad 6x - 3y \cancel{- z} = 35 \\ \hline &④ \quad 8x - 2y = 50 \end{aligned}$$

$$\begin{aligned} &① \quad 2x + y \cancel{+ z} = 15 \\ ⊕\ &③ \quad -4x + 4y \cancel{- z} = -14 \\ \hline &⑤ \quad -2x + 5y = 1 \end{aligned}$$

(b) Use this new two-by-two system to solve the three-by-three.

$$8x - 2y = 50$$
$$4(-2x + 5y = 1) \rightarrow$$

$$\begin{aligned} ⊕\ & \cancel{8x} - 2y = 50 \\ & \cancel{-8x} + 20y = 4 \\ \hline & 18y = 54 \\ & y = 3 \end{aligned}$$

To find x: Use Equation ④ or ⑤

$$8x - 2(3) = 50$$
$$8x - 6 = 50$$
$$\underline{\quad +6 \quad +6}$$
$$8x = 56$$
$$x = 7$$

To find z: Use equation ①, ② or ③

$$2x + y + z = 15$$
$$2(7) + 3 + z = 15$$
$$14 + 3 + z = 15$$
$$17 + z = 15$$
$$z = -2$$

$x = 7$
$y = 3$ OR $(7, 3, -2)$
$z = -2$

Just as with two by two systems, sometimes three-by-three systems need to be manipulated by the **multiplication property of equality** before we can eliminate any variables.

8. Consider the system of equations shown below. Answer the following questions based on the system.

$$\begin{aligned} ① \quad 4x + y - 3z &= -6 \\ ② \quad -2x + 4y + 2z &= 38 \\ ③ \quad 5x - y - 7z &= -19 \end{aligned}$$

(a) Which variable will be easiest to eliminate? Why? Use the multiplicative property of equality and elimination to reduce this system to a two-by-two system. ① and ③

$$\begin{aligned} ① \quad 4x \cancel{+y} - 3z &= -6 \\ ③ \oplus \quad 5x \cancel{-y} - 7z &= -19 \\ \hline ④ \quad 9x - 10z &= -25 \end{aligned}$$

$$\begin{aligned} ② \quad -2x + 4y + 2z &= 38 \\ ① \quad -4(4x + y - 3z &= -6) \\ -16x \cancel{-4y} + 12z &= 24 \\ \oplus \quad -2x \cancel{+4y} + 2z &= 38 \\ \hline ⑤ \quad -18x + 14z &= 62 \end{aligned}$$

(b) Solve the two-by-two system from (a) and find the final solution to the three-by-three system.

$$2(9x - 10z = -25)$$
$$-18x + 14z = 62$$

$$\begin{aligned} \cancel{18x} - 20z &= -50 \\ \oplus \quad \cancel{-18x} + 14z &= 62 \\ \hline -6z &= 12 \\ z &= -2 \end{aligned}$$

To find x:

$$\begin{aligned} -18x + 14(-2) &= 62 \\ -18x - 28 &= 62 \\ +28 \quad & \quad +28 \\ \hline -18x &= 90 \\ x &= -5 \end{aligned}$$

To find y: use ①

$$\begin{aligned} 4(-5) + y - 3(-2) &= -6 \\ -20 + y + 6 &= -6 \\ -14 + y &= -6 \\ y &= 8 \end{aligned}$$

$-5, 8, -2$

$x = -5$
$y = 8$
$z = -2$

9. Solve the system of equations shown below. Show each step in your solution process.

$$
\begin{aligned}
4x - 2y + 3z &= 23 \\
x + 5y - 3z &= -37 \\
-2x + y + 4z &= 27
\end{aligned}
$$